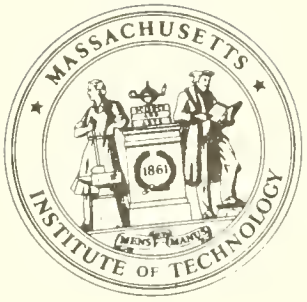

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CASH MANAGEMENT: A SYSTEMS APPROACH*

by

G. A. Pogue,[1] R. B. Faucett,[2] R. N. Bussard[3]

Revised November 1969

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INTRODUCTION

Corporations maintain cash balances for essentially three reasons. Their inflow and outflow of cash payments are not perfectly synchronized; therefore, an inventory of cash is necessary to buffer these flows. Forecasts of cash flows and of interest rates are uncertain; therefore, corporations hold cash to insure themselves against unfortunate future events in their cash flows and in the fluctuations of interest rates. Thirdly, corporations use cash balances to compensate their banks for a variety of services that may be provided. This article focuses on the problem of determining the optimal level of cash balances needed to support a firm's banking system. The model described below determines the optimal level and inter-bank allocation of cash balances at which the total cost of the banking system is minimized. Total cost includes the opportunity cost of cash balances as well as any fees paid directly to the banks. In addition to determining the optimal allocation of cash balances, the model determines the best allocation of the firm's checking and deposit activity within the system. As will be discussed later, the model is also extremely useful for evaluating proposed changes in the structure of the firm's banking system (e.g. adding or deleting banks).

The general approach to this problem was developed by Robert Calman,[1] treasurer of the International Division of the Mobil Oil

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Corporation. In this article we present a brief description of an extended version of Calman's model and an application to the cash management problem of a large eastern corporation.[1]

Bank Services

The services that banks provide to corporations can be separated into two categories, tangible and intengible services. Tangible services primarily include disbursement and payroll accounts, collection of deposits, stock registration and transfer, current lines of credit or term loans, and retail and wholesale lock boxes.[2] Probably the most important intangible service is the call on future

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[1] Readers interested in a detailed discussion of the underlying model should refer to Calman's book, Linear Programming and Cash Management/CASH ALPHA, M.I.T. Press, Cambridge, Massachusetts, 1968.

[2] Lock Boxes are Post Office Boxes maintained by banks for corporate customers in the geographic areas where company customers are located. Customers mail their remittances to a designated lock box rather than to a more distant company accounting center where they are collected and processed directly by the bank responsible for the box. Essentially, Lock Box Banking speeds up cash inflow by reducing the number of stages and days needed for a company to see the tangible balance-sheet benefit of its customers' payments. It does this by shortening three things: the mail time between the point of mailing by the company's customer and the actual receipt of the check; the delay in processing the mail and depositing remittances in the bank; and the "in transit" clearing time within the banking system between the bank of deposit and the bank on which the check was drawn.

credit from the bank. By maintaining high cash balances a corporation can insure the future availability of credit from its banks. Other intangibles could include special work analogous to that provided by an independent management consulting firm. For example, this might include advice on mergers, or international business, or on economic conditions. Intangibles might also include any special services that the bank provides to corporate employees.

Banks are compensated for their services by fees, compensating cash balances, and by tax payments. Fees have traditionally been the form of compensation for stock certificate registration and transfer. Recently, some banks have distributed fee schedules for compensation of wholesale and retail lock box services. To a lesser extent, banks have distributed fee schedules for other tangible services such as checking, collection of deposits, and payroll. Traditionally banks have been compensated by cash balances that corporations kept in their demand accounts. Typically, the bank computes a service charge credit on available-for-investment balances that offsets a part or all of the cost of services provided. The third alternative is to compensate the bank by directing the corporate tax payments into a "Tax and Loan Account" at the bank. Often a bank will compute a service charge credit on the balances in this account because the U.S. Treasury usually does not withdraw funds from this account until eight to ten days after the corporation has been required to deposit those funds into the account.

Usually fees are restricted as compensation only for the tangible services related to checking, deposit, and payroll activity and to stock certificate activity. Cash balances can be used as compensation for all

services, tangible and intangible, that the banks provide. Compensating balances are doubly useful: they can be used as compensation for bank services, while they simultaneously compensate for bank credit. Tax payments usually are analogous to cash balances in that they can be used as compensation for all services provided. Since demand deposits are a primary source of funds for investment, many banks are reluctant to use compensation plans that do not produce large balances in their demand deposit accounts. Thus a bank usually allows its corporate customers to use fees and tax payments as only a fraction of the total compensation.

To determine the proper level of compensation to a bank is one of the more awkward problems of cash and bank management for a corporation. Traditionally, banks have been extremely reluctant to release price schedules for their services. Increased competition between banks for lock box business has produced schedules for that service. In some instances banks have released schedules for the tangible services of checking, deposits, and payroll to a customer. However, even in cases where these schedules have been released, more often than not, they have been "cost" schedules rather than prices. Banks have continued to allow their customers to determine the proper level of compensation for services provided. Banks, however, are no different from other service organizations in the economy, especially with respect to the tangible services they provide. Thus, there is no reason why banks should not supply their customers with price schedules for the services that they provide.

The general model developed by Calman and the application described here assumes that management, in collaboration with its various banks, has determined prices for tangible services and that management has estimated the approximate level of compensation for intangible services. Thus, the prices, not costs, of processing a check, a deposit or a payroll check are known. The level of compensating cash balances required by any current credit agreement and the service charge credits given on cash balances are known. Furthermore, the value to the corporation of intangibles has been estimated by some rule of thumb. For example, to insure the availability of future credit, management in the case described below assumed that it was sufficient to maintain a cash balance equal to 5% of any future amount they wished to borrow from a bank. The value of other intangibles, such as advice on international business, was estimated by determining the price of those intangibles as if they had come from another source that actually charged for the service, for example, a management consulting firm.

Disbursement and Deposit Float

Generally, the amount of funds available to a company in its bank account will be different than the book cash balances indicated on the cor-

poration's ledger. This difference between the bank's "net collected balance" and corporate book balance is the cumulative result of a series of delays in the payment, by the firm's banks, of checks written by the company (for which the corporate book balances have been reduced) and the actual collection of checks received by the company (for which corporate ledger balances have already been increased). The difference is generally referred to as float. Checks written by the company will generally result in "positive float," which is an excess of bank net collected balances over corporate book balances. Conversely, checks received by the company and deposited in the banking system tend to result in "negative float" or an excess of book balances over bank net collected balances. These concepts will be illustrated below.

Exhibit 1 illustrates the principal sources of disbursement float. The float is caused by 1) the mailing time to the firm's creditor; 2) the creditor's delay in processing the check; and 3) the normal check collection time in the banking system.

Exhibit 2 illustrates the relationship between corporate book balances, bank balances and disbursement float balances. In the example it is assumed that the corporation disburses at a rate of one million dollars per day, and a five-day delay exists before checks are presented to the firm's bank for payment. This results in a positive float balance of 5 million dollars.[1] In other words, five days worth of checks are always in the "payment pipeline" illustrated in Exhibit 1. Now if the company immediately transfers cash to its bank account form its marketable securities portfolio when checks are written, its

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[1] Assuming steady state conditions.

EXHIBIT 1

SOURCES OF DISBURSEMENT FLOAT

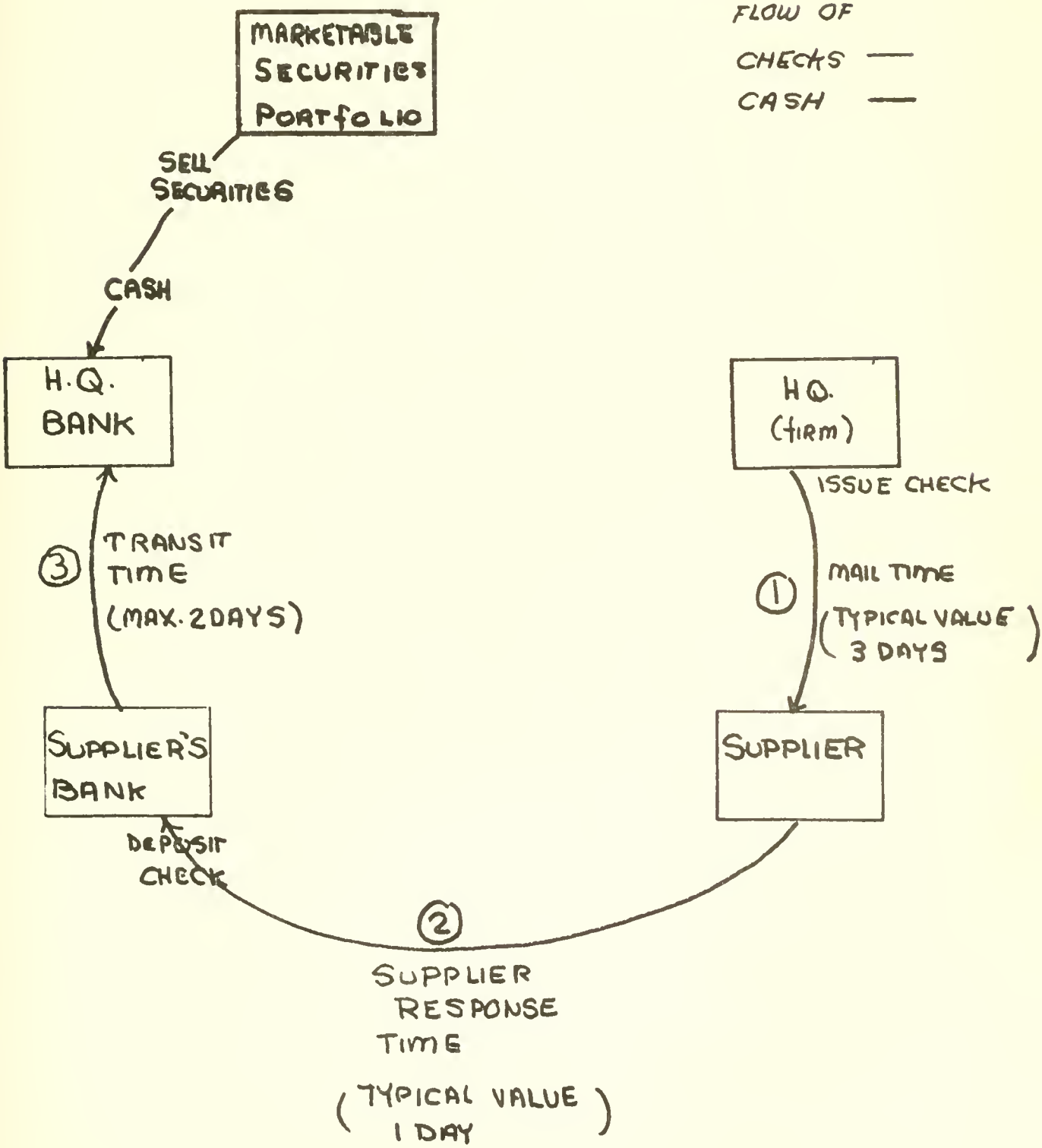

Exhibit 2

CHECK FLOAT: ILLUSTRATIONS

ASSUME: 1. Firm issues 1 million dollars of checks per day.

2. A 5-day delay in presentation of checks for payment

3. Cash is transferred to bank some number of days after issue of check.

CASH TRANSFER DELAY (DAYS AFTER CHECK ISSUE)	FIRM BOOK BALANCE	FLOAT BALANCE ($)	BANK NET COLLECTED BALANCE ($)
0	0	5	5
1	-1	5	4
2	-2	5	3
3	-3	5	2
4	-4	5	1
5	-5	5	0

book cash balance will always be identically zero. Its bank balance, however, will equal 5 million dollars. (See Line 1 of Exhibit 2). An interesting possibility now becomes evident. By delaying the transfer of cash from the marketable securities portfolio to its bank account for some number of days, the firm can continue to earn a return on those funds, while simultaneously maintaining a desired positive level of bank net collected balances. For example, if the firm delayed the transfer for three days, an average bank balance of 2 million dollars would result (see Line 4 of Exhibit 2). In effect, the firm is maintaining a negative book balance by investing part of the disbursement float. Now, in a practical situation, the firm will not be able to predict the size of float balances exactly, as in our simple example, so that more caution must be taken in the utilization of expected float balances. More will be said about this problem later.

As might be expected, deposit float, unlike disbursement float, works to the disadvantage of the corporation. Exhibit 3 illustrates the main stages involved in the cash collection process. The volume of deposit float is proportional to the time interval between the time when the corporation records the incoming check on its ledgers and the time the funds physically become available in its bank account. (Stages 2 & 3 on Exhibit 3) A further collection delay indicated in Exhibit 3 is the mail time between the customer's location and the company's offices. This delay tends to manifest itself in accounts receivables balances rather than deposit float. However, through the use of a lock box, the "in mail" and "in transit" collection times could both potentially be reduced, the former reducing the firm's investment in receivables balances, the later reducing the investment in deposit float balances.

EXHIBIT 3

SOURCES OF DEPOSIT FLOAT

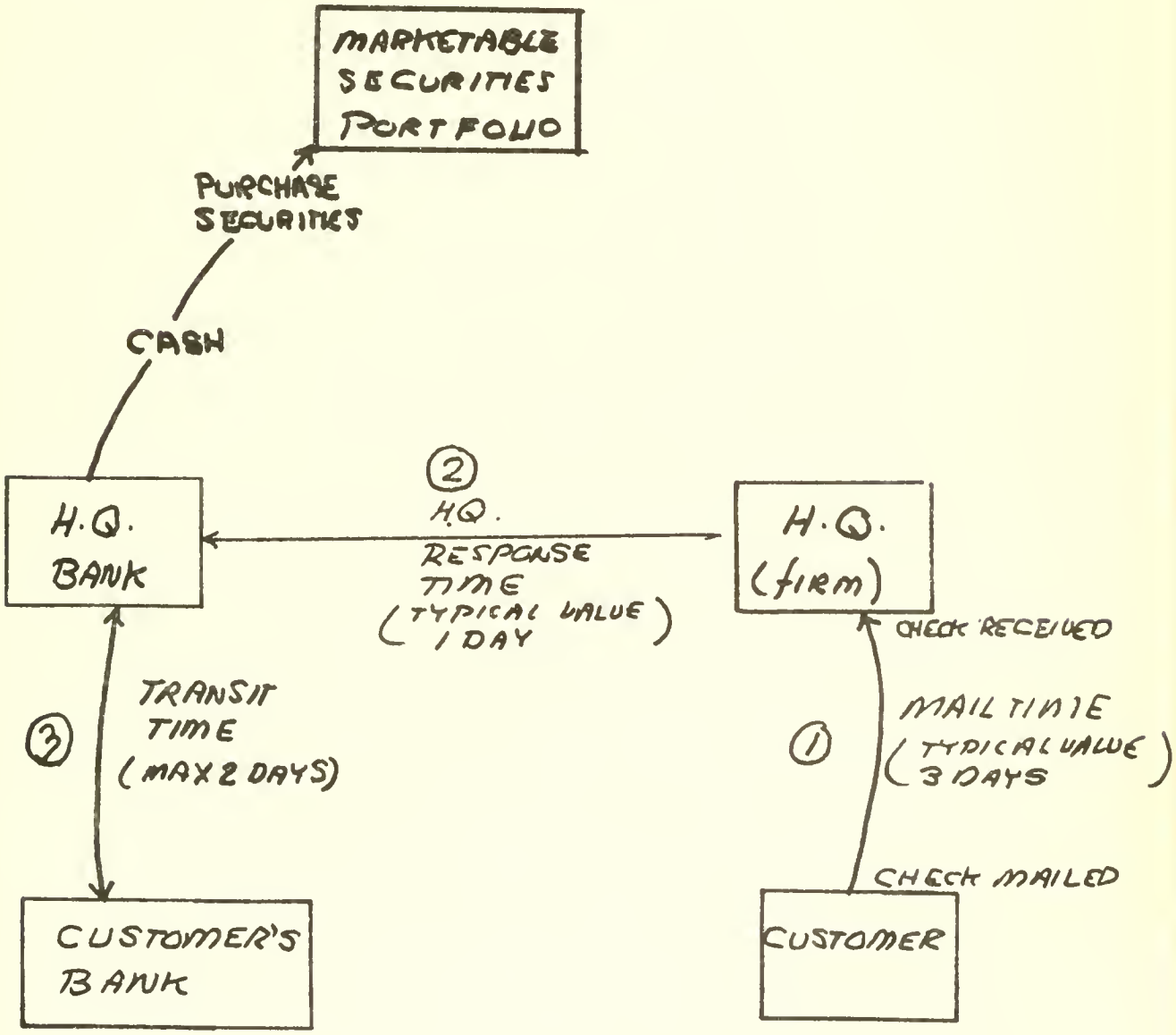

Exhibit 4 provides an illustration of the relationship between corporate, bank and deposit float balances. The example assumes that the company receives customer remittances at the rate of one million dollars of checks per day and these funds are transferred to the marketable secueities portfolio immediately after collection by the firm's bank. This policy, as shown, would result in a zero net collected balance and a ledger cash balance equal to the dollar value of deposit float. Thus, rather than being able to invest part of the float and still maintain positive bank balances, as in the disbursement case, the firm now must maintain a cash balance in excess of the amount of deposit float if it wishes to maintain a positive net collected balance.[1] In a more normal situation, where both disbursement and deposit activities are carried on, both types of float will exist simultaneously and the net float is the important factor.

As evident from the previous discussion, the net float balance resulting from a given dollar rate of disbursements or collections are due to the cumulative effects of a number of payment or collection delays, none of which in practice can be predicted exactly. In the model described below we have attempted to deal with this prediction problem by treating the float balances generated by particular types of corporate activities as random variables. As such, it then is necessary to evaluate the float characteristics of each major disbursement or deposit activity separately.

---

[1] Of this amount, only the excess of ledger balances over deposit float balances will appear as cash balances in the firm's account.

Exhibit 4

DEPOSIT FLOAT: ILLUSTRATIONS

ASSUME: 1. Firm receives checks at rate of one million dollars per day

2. A 5-day collection delay

3. Cash is transferred to marketable securities portfolio only after it has been collected by the bank

COLLECTION DELAY (DAYS AFTER RECEIPT)	TRANSFER DELAY	FIRM BOOK BALANCE ($)	DEPOSIT FLOAT ($)	BANK NET COLLECTED BALANCE ($)
0	0	0	0	0
1	1	1	1	0
2	2	2	2	0
3	3	3	3	0
4	4	4	4	0
5	5	5	5	0

For example, the float characteristics per dollar of dividend checks issued may be quite different from those for vendor or payroll checks.

The probability distribution for a particular type of disbursement float can be easily estimated from the collection data stamped on the backs of previously processed disbursement checks. The "in transit" time for deposit items in effect is known with certainty if the firm's bank enters its checks into the Federal Reserve System for collection.[1] The funds are automatically available after a predetermined number of days depending on the geographic proximity of the firm's and customer's banks.[2]

The model developed below is an extension of the original Cash Alpha Model. Our extended version permits the allocation of negative book balances to banks in the system where favorable float characteristics and other considerations permit.[3] Without this extension, the firm could be in the undesirable position of having net collected balances in excess of the amounts needed to adequately compensate its system of banks. Given the possibility of negative corporate ledger balances, it now becomes vital to consider the amount of uncertainty associated with projected float balances. For banks where only small average collected balances are to be

---

[1]This is the normal procedure.

[2]The maximum number of days is equal to 2.

[3]The use of negative book balances is a reasonably common industrial practice.

maintained, meglect of the random nature of future float balances, could lead to a high frequency of overdrafts (i.e. negative net collected balances). A high frequency of overdrafts could produce unfavorable bank reactions which would damage the credit rating of the company. In our version of Cash Alpha we have extended Calman's expected value treatment of float to consider the random character of the float coefficients explicitly.

Description of the Model

Once the prices of tangible services, the inputed prices of intangible services and the float characteristics of the firm's various checking and deposit activities have been established, the problem of allocating cash balances and levels of activity in order to minimize the cost of the firm's banking system can be formulated as a linear programming problem.

The objective of the model is to minimize the sum of fees paid and the opportunity cost of cash balances allocated to the banks in the system. This is accomplished by allocating tangible activities (such as checks, deposits, tax payments, etc.) and cash ledger balances to banks such that each bank will be adequately compensated for all services the firm expects to receive during the period under consideration (such as the next month) at the lowest total cost to the firm. The cost of cash balances is the return that cash would yield if invested in an alternative use of the same riskiness. For most corporations the alternative use would be investment in short-term securities. In that case, the cost of cash allocated to the

banking system would be approximately 7 to 8 percent per year.[1]

The primary decision variables in the model are defined below.

BAL_k = the average company ledger cash balance assigned to bank k during period[2],

where k = 1, . . ., kmax

where kmax = the number of banks in the firm's banking system.

FEE_k = Amount of fees to be paid to bank k during the period.

X_{ijk} = the disbursing activity variable

= the number of checks to be issued during the period ,

(i) by financial center i (i.e. by the various company divisions), where i = 1, . . ., imax;

(ii) of type j (vendor, payroll tax etc.), where j = 1, . . ., jmax;

(iii) on bank k, where k = 1, . . ., kmax.

---

[1]Two additional assumptions regarding company cash ledger balances are implicit in the model.

(i) The firm's cash ledger balances are decreased or increased at the time when checks are issued or received by the corporation and not when the funds are paid out or collected by the banks. (The usual practice.)

(ii) Disbursement float is assumed to have a zero direct cost to the firm. (However, it does have an indirect opportunity cost since, as seen in the previous examples, a dollar of ledger balances can be replaced by a dollar of disbursement float balances with no effect on the level of net collected balances.)

[2]In the linear programming formulation actually used, we have replaced the variable BAL_k by the pair of variables BAL_k^+ and BAL_k^- where $BAL_k = BAL_k^+ - BAL_k^-$. This addition allows the ledger cash balance to take negative as well as positive values.

Y_{imk} = the deposit activity variable

= the number of deposit items during the period

(i) processed by financial center i, where i = 1, . . ., Imax;

(ii) of type m (e.g. customer location), where m = 1, . . ., mmax;

(iii) sent to bank k for collection, where k = 1, . . ., kmax.

The objective function is then given by

Minimize Z = (BAL) + FEE

where = the opportunity cost of ledger cash balances

BAL = Total cash ledger balance of corporation

FEE = Total amount of fees paid by company.

This minimization is subject to certain constraints. These constraints may be separated into two categories, global constraints and individual bank constraints. The global constraints are the following:

1) The total cash ledger balance of the system is the sum of the cash ledger balances associated with each bank.

$$BAL = \sum_{k=1}^{kmax} BAL_k$$

2) The total amount of fees paid is the sum of the fees paid to each bank.

$$BAL = \sum_{k=1}^{kmax} FEE_k$$

3) The total number of checks and deposits allocated to banks by each financial center must equal the expected number available. Thus for financial center i we have

$$\sum_{k=1}^{kmax} X_{ijk} = \hat{X}_{ij} \quad , \qquad j=1, \ldots, jmax$$

$$\sum_{k=1}^{kmax} Y_{imk} = \hat{Y}_{im} \quad , \qquad m=1, \ldots, mmax \ .$$

where

$\hat{X}_{ij}$ = the expected number of checks of type j that will be issued by financial center i in the next month.

$\hat{Y}_{im}$ = the expected number of deposits of type m that will be processed by financial center i in the next month.

In the application which will be described below, the firm has four financial centers. For the month of May 1969 the following were the activity level projections are shown in Table I.[1] Thus, the above financial center requirements results in twelve equality constraints.

Table I

PROJECTED FINANCIAL CENTER ACTIVITY LEVELS

Center Number i	Financial Center Name	Expected Number of Vendor Checks During Month $\hat{X}_{i1}$	Expected Number of Payroll Checks During Month $\hat{X}_{i2}$	Expected Number of Deposit Items During Month $\hat{X}_{i1}$
1	Headquarters	2600	8500	475
2	Quaker Industrial Products	600	1900	20
3	Progressive Business Products	2400	4300	5800
4	California Engineering	1400	2200	20

[1]The names of company divisions and banks in the system have been disguised.

The individual bank constraints describe the relationships between a specific bank and the company. These constraints will be described using a specific bank, New England Citizens, from the banking system in the application. This bank processes vendor disbursements, deposits and payroll checks for two of the firm's divisions (Headquarters and California Engineering), extends to the company a line of credit and provides other intangible services, mainly advice on international financial conditions. There is a similar set of constraints for each other bank in the system.

1. Expected Float Balances Definitional Constraint[1]

The expected average disbursement float balances during the month associated with each bank, are the sum of the float balances arising from the various checks drawn on the bank.

$$\overline{\text{PFLOAT}} = \sum_{i=1}^{\text{imax}} \sum_{j=1}^{\text{jmax}} \overline{\text{PFLOAT}}_{ij} \ .$$

The above is equal to the total dollars disbursed on the bank times the expected number of days (expressed as a fraction of a month) from the time a particular financial center issues a specific type of check until the check arrives at the bank for payment

$$\overline{\text{PFLOAT}} = \sum_{i=1}^{\text{imax}} \sum_{j=1}^{\text{jmax}} X_{ij} \left(\begin{array}{l} \text{Average} \\ \text{Check} \\ \text{Value;} \\ \text{Center i,} \\ \text{Type j.} \end{array} \right) \left(\begin{array}{l} \text{Expected} \\ \text{Days pay-} \\ \text{ment Delay;} \\ \text{Center i} \\ \text{Type j} \\ \hline \text{Average No.} \\ \text{Days in} \\ \text{Month} \end{array} \right) .$$

[1] To simplify notation, the subscript k which identifies the bank has been dropped from all decision variables and parameters for the remainder of this section.

Table II

BASIC BANK DATA

NEW ENGLAND CITIZENS BANK

Item	Value
1. Net collected balance credit (rate per month)	0.002
2. Charge per vendor check (dollars per item)	0.04
3. Charge per deposit (dollars per item)	0.02
4. Charge per payroll check (dollars per item)	0.06
5. Fixed charges (dollars per month)	0.0
6. Value of intangible services (dollars per month)	1500
7. Line of credit (dollars)	4,000,000
8. Compensating balance (dollars per dollar)	0.20
9. Expected increase in line of credit (dollars)	0.0
10. Supporting balance (dollars per dollar)	0.05

Table III

FINANCIAL CENTER-BANK DATA

NEW ENGLAND CITIZENS BANK

Item	Value	
	Headquarters Division	California Engineering
Average size of vendor check	$ 1,100	$ 1,300
Expected days disbursement float	6.6	8.3
Desired cushion on vendor checks*	0.10	0.15
Average size of payroll check	$ 165	**
Expected days disbursement float	2.0	**
Desired cushion on payroll checks	0.60	**
Average size of deposit item	$12,000	$95,000
Expected days deposit float	1.0	2.0
Desired cushion on deposit items	0.20	0.40

* Defined below

** Did not issue payroll checks on New England Citizens Bank

For the New England Citizens Bank

$$\overline{PFLOAT} = 240X_{11} + 11X_{12} + 360X_{41} \quad .$$

Similarly, the expected average monthly deposit float balance (negative float) is proportional to the total dollars of deposits and the expected in transit collection time for the New England Citizens Bank.

$$\overline{NFLOAT} = 400Y_{1} + 6{,}300Y_{41} \quad .$$

2. Expected Net Collected Balance Definitional Constraint

The average monthly ledger balance plus the expected deposit float balance, minus the expected deposit float balance equals (by definition), the expected average net collected balance during the month.

$$BAL + \overline{PFLOAT} - \overline{NFLOAT} = \overline{NCB} \quad .$$

3. Cost of Expected Tangible Bank Services During the Month

The cost of processing vendor and payroll checks is given by

$$CHCST = \sum_{i=1}^{imax} \sum_{j=1}^{jmax} X_{ij} d_j \quad ,$$

where d_j = dollar cost per item of type j .

For the New England Citizens Bank

$$CHCST = 0.04X_{11} + 0.06X_{12} + 0.04X_{41} \quad .$$

Similarly the deposit cost is given by

$$DCST = 0.02Y_{11} + 0.02Y_{41} \quad .$$

4. Bank Compensation Constraint

The total compensation paid to the bank must be at least equal to the cost of expected bank services.

$$\text{FEE} + 0.002\,\overline{\text{NCB}} \geq \text{CHCST} + \text{DCST} + \begin{matrix}\text{FIXED}\\ \text{CHARGES}\end{matrix} + \begin{matrix}\text{VALUE}\\ \text{INTANGIBLE}\\ \text{SERVICES}\end{matrix} + \text{MAKEUP} \quad ,$$

where MAKEUP = charges based on any under or over compensation of the bank during previous periods;

FIXED CHARGES = \$0.0;

VALUE INTANGIBLE SERVICES = \$1500.

5. FEE Limitation Constraint

The New England Citizens Bank limits the use of fees to 20% of the total compensation for tangible services.

$$\text{FEE} \leq 0.20\,[\text{CHCST} + \text{DCST} + 1500 + \text{MAKEUP}] \quad .$$

6. Net Collected Balance Constraint -- Compensating Balances

The New England Citizens Bank requires the average net collected balance to be greater than 20% of any existing line of credit. In addition, the company officers felt that to insure future borrowing capability, 5% of any future requirements should be maintained in current bank balances.

$$\begin{aligned}\overline{\text{NCB}} &\geq \begin{matrix}\text{COMPENSATING}\\ \text{BALANCE}\\ \text{REQUIREMENT}\end{matrix} + \begin{matrix}\text{SUPPORTING}\\ \text{BALANCE}\\ \text{REQUIREMENT}\end{matrix}\\ &= 0.20(4{,}000{,}000) + 0.05(0.0)\\ &= 800{,}000 \quad .\end{aligned}$$

7. Net Collected Balance Constraint -- Float Variation

The expected net collected balance must also be greater than an amount which is related to the uncertainties in the float values.

To some extent all of the actual float values are uncertain quantities and will differ during the coming period from the expected values.[1] We require that the probability that the actual average net collected balance during the coming period be negative (i.e. in overdraft) be less than a specified amount, say 2%. This constraint prevents the account from being frequently overdrawn due to an abnormally low disbursement float or high deposit float. In the model the desired "float cushion" for expected net collected balances is a percentage of the sum of the expected disbursement, payroll and deposit floats.

For the New England Citizens Bank, the constraint is given by

$$\overline{NCB} \geq 0.10\ \overline{PFLOAT}_{11} + 0.60\ \overline{PFLOAT}_{12} + 0.15\ \overline{PFLOAT}_{41} + 0.20\ \overline{NFLOAT}_{11} + 0.40\ \overline{NFLOAT}_{41} .$$

The derivation of the constraint is discussed in Appendix A.

THE APPLICATION

The authors have applied this general model to a large corporation (sales are $100 million). The corporation is organized into five divisions that are located across the country. The growth of the company has come

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[1] Deposit float balances are treated as random variables, not because of any uncertainty in the collection time for a specified item, but due to uncertainty associated with item size and the mix of deposit item types. Some of the uncertainty due to the item mix problem can be eliminated by defining more homogenous item classes. This would increase the number of deposit type categories and hence the size of the linear programming problem.

primarily from acquisition. As each division was acquired, it added more banks to the corporate banking system. At the time of the study, each division had at least two local banks which it used for disbursements, deposits, and payroll. In addition, the corporation had a $10 million line of credit from the three largest banks.

Corporate management believed that there were too many banks in the system and that their cash balances were not being used effectively. The line of credit agreement required a substantial compensating balance at each of the three large banks. Corporate management was aware that substantial amounts of disbursement float were available in the system. Management was anxious to use these floats, to the extent possible, to generate the required compensating balances, and then to maximize the use of these balances as compensation for other services. At the same time management recognized the need to maintain local banks to serve the divisions, primarily for payroll and various intangible services.

By using the above approach to structure this company's banking system, the authors were able to demonstrate the total cash ledger balance in the system could be reduced from $1.6 million to $437,000 and that the total monthly banking cost could be reduced from $8,700 to $3,600. These reductions primarily came from a more effective use of disbursement float and the re-allocation of tangible services to banks that had to have substantial net collected balances.

Exhibit 5 illustrates the flow of information and decisions within the cash management system. This exhibit attempts to describe how the various components of the cash management system are interrelated. The

Exhibit 5

Cash Management Model
INFORMATION AND DECISION FLOW DIAGRAM

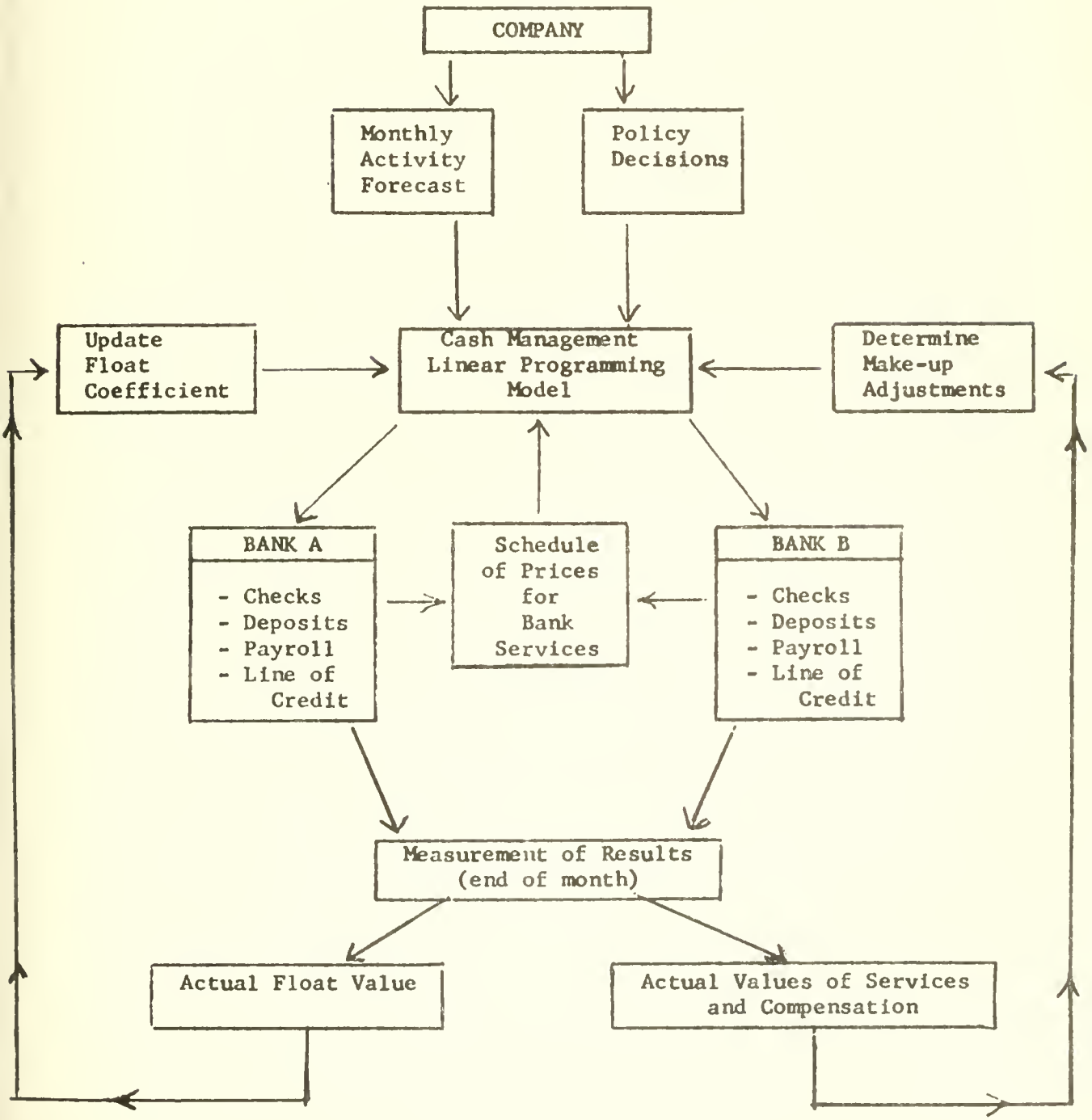

Exhibit 6

Comparison of Banking System Activities
Before and After Application of Cash Management Model

Name of Bank	Book Balances (Dollars)		Net Collected Balances (Dollars)		Fees Paid (Dollars)		Check Activity Allocation (incl. payroll) (Dollars)	
	Before	After	Before	After	Before	After	Before	After
1. New England Citizens' *	NA	-276,039	648,000	800,000	56	579	1,820,000	6,214,000
2. Empire City Bank*	NA	872,666	625,000	600,000	0	0	1,156,000	0
3. Pacific Southern Trust*	NA	559,400	700,000	600,000	0	0	2,121,000	580,000
4. Pilgrim State Bank	NA	30,000	150,000	30,000	450	0	568,000	0
5. Steeltown Federal Bank	NA	-608,750	160,000	84,750	0	281	4,440,000	3,170,000
6. Third National Bank of Pittsburgh	NA	-134,400	90,000	14,933	0	18	183,000	640,000
7. First American Trust	NA	-5,740	350,000	5,740	0	103	480,000	164,000
TOTALS	1,610,000	437,134	2,723,000	2,135,420	506	981	10,768,000	10,768,000

*A Line of credit totaling 10 million dollars is provided jointly by these banks.

Bank locations are: Massachusetts (1,4), New York (2,5), Pennsylvania (6,7), California (3).

Exhibit 7

SAMPLE COMPUTER PRINTOUT FOR ONE OF THE BANKS IN
THE CASH MANAGEMENT SYSTEM

New England Citizens' Bank

	NCB REQUIRED
BINDING NCB REQUIREMENT:	
0.20 COMPENSATING BALANCE FOR EXISTING $4000000. LOAN PLUS	
0.05 SUPPORTING BALANCE FOR FUTURE $ 0. LOAN	800000.
NON-BINDING REQUIREMENTS:	
NET COLLECTED BALANCE GREATER THAN ZERO	0.
PAY FOR $ 1500./MONTH INTANGIBLE SERVICES	750000.
FLOAT RESERVED FOR FLOAT VARIATION	238388.

BOOK BALANCE	-276039.	FIXED MONTHLY CHARGE	0.
NET FLOAT	1076038.	CHECK CHARGE (0.04 EA)	160.
TOTAL NCB CREDIT (0.002/$NCB)	1600.	DEPOSIT CHARGE (0.02 EA)	9.
TOTAL FEE PAYMENT	579.	PAYROLL CHARGE (0.06 EA)	510.

	CENTER NAME	AVG DAYS FLOAT	% CUSHION	TOTAL DOLLARS
CHECK ACTIVITY:	HEADQUARTERS	6.6	0.1000	2949996.
	CALIFORNIA ENGINEERING	8.3	0.1500	1863995.
DEPOSIT ACTIVITY:	HEADQUARTERS	1.0	0.2000	5600000.
	CALIFORNIA ENGINEERING	2.0	0.4000	0.
PAYROLL ACTIVITY:	HEADQUARTERS	2.0	0.6000	1399997.

system is an adaptive type of feedback structure, where differences between planned and actual results in the previous month, plus expectations regarding banking activities in future months, are considered by the linear programming model in developing its recommendations for allocation of the projected next months' banking activity. When used as an operating tool, the model optimally allocates activities through a predetermined banking structure and according to a specified set of corporate policies. In addition, however, the model can be an extremely valuable planning tool for use in the design or modification of the corporation's banking structure and policies.

Exhibit 6 presents a comparison of how book balances, net collected balances, fees and checking activity were allocated in the existing system during a given month with the allocation recommended by the cash management model.

Exhibit 7 is a sample printout for one bank in the system. This printout summarizes the level of activity at the bank and the way in which the bank is being compensated. The "BINDING NCB REQUIREMENT" refers to that constraint responsible for maintaining the current level of net collected balances at the bank. If that constraint were relaxed then the computer program would be able to lower further the book balance associated with that bank.

The binding constraint on net collected balances is only one indication of tightness in the banking system. The linear programming routine also computes the marginal cost of additional dollars of activity at each bank. Management can use these marginal costs as indications of banks or

bank services whose prices should be renegotiated. The system approach used here is particularly useful for evaluating the results of a price renegotiation. It is important to remember that renegotiation not only lowers the cost of banking at a particular bank; it can also cause a re-allocation of business from other banks in the system to that bank. This re-allocation could substantially reduce the activity levels and cash balances allocated to other banks in the system. Without the system approach of the linear programming model, most of these re-allocations and system savings could easily be overlooked.

In addition to using the marginal prices and the binding constraints as indications for negotiation, they can also be used to indicate the need for structural change in the banking system. In the application discussed here, it was found that one bank in the line of credit agreement was quite expensive. The expense was not the result of an excessive price structure, but primarily due to the location of the bank relative to other banks in the system and the corporate divisions which it served. The model determined that sufficient activity could not be allocated to this bank to use up the substantial net collected balance credits made available via the compensating balance requirement. The management is currently considering the alternative of replacing this bank by one which could more effectively be part of the overall system. The effect on the system is being evaluated by introducing various candidate banks into the linear programming model and evaluating the resulting solutions.

The linear program also can evaluate potential lock box locations. Many banks provide lock box location assistance. Generally, this assistance focuses on the problem of minimizing deposit float while keeping lock box fees within reasonable size. However, lock boxes are only one

part of the banking system, and they should not be evaluated separately. The existence of compensating balances at certain banks will make it more profitable to allocate some lock box activity at those banks than at others, even though other banks could further lower the deposit float. This allocation can be made correctly only in the context of the other variables in the system. By adding constraints that compute lock box fees, the float, and the allocation of lock box items, the linear programming model can evaluate the potential lock box sites.

SUMMARY

Cash management requires decision-making in several areas: the use of balances or fees for compensation; the allocation of specific activities to specific banks; the location and optimal usage of lock boxes; and the use of disbursement float as a substitute for permanent balances. Making these decisions sequentially without considering the impact on other banks or other activities can result in costly sub-optimization. Linear programming can be used to optimize the cost of the total system subject to management policy and operating constraints.

Appendix A

Development of Float Variation Constraint

The actual average net collected balance during the future period is a random variable, due to its relationship to the average float balances which are random variables. We have

$$\widetilde{NCB} = BAL + \widetilde{PFLOAT} - \widetilde{NFLOAT} \tag{1a}$$

$$= BAL + \sum_i \sum_j \widetilde{PFLOAT}_{ij} - \sum_i \sum_m \widetilde{NFLOAT}_{im} \quad . \tag{1b}$$

The required constraint on actual average net collected balances is given by

$$P(\widetilde{NCB} \leq 0) \leq \epsilon \, , \tag{2}$$

where ϵ is, for example, 2%.

Now relationship (2) is equivalent to

$$P\left[\frac{\widetilde{NCB} - \overline{NCB}}{\sigma(\widetilde{NCB})} \leq \frac{-\overline{NCB}}{\sigma(\widetilde{NCB})}\right] \leq \epsilon \, , \tag{3}$$

where $\sigma(\widetilde{NCB})$ = Standard Deviation of $\widetilde{NCB}$

$\overline{NCB}$ = Expected Value of $\widetilde{NCB}$.

By Tchebyscheff's extended lemma we have[1]

$$P\left(\frac{\widetilde{NCB} - \overline{NCB}}{\sigma(\widetilde{NCB})} \leq W\right) \leq \frac{1}{1+W^2}$$

or $$P(\widetilde{NCB} \leq \overline{NCB} + W\sigma(\widetilde{NCB})) \leq \frac{1}{1+W^2} \, , \tag{4}$$

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[1]Harold Cramer, Mathematic Methods of Statistics, Princeton University Press (1946) P. 256, Exercise 5.

where $w < 0$.

Now, if we take

$$\epsilon = \frac{1}{1+W^2}$$

from which $w = -\left[\frac{1-\epsilon}{\epsilon}\right]^{1/2}$

then any $\widetilde{NCB}$ satisfying

$$\overline{NCB} + W(\sigma(\widetilde{NCB})) \geq 0 \quad , \tag{5}$$

where $$-\frac{\overline{NCB}}{\sigma(NCB)} \leq W < 0 \quad ,$$

will also satisfy the original probability constraint. From expression 1(b) and the properties of standard deviations it follows that

$$\sigma(\widetilde{NCB}) \leq \sum_i \sum_j \sigma(\widetilde{PFLOAT_{ij}}) + \sum_i \sum_m \sigma(\widetilde{NFLOAT_{im}}) \quad . \tag{6}$$

Now if we replace $\sigma(NCB)$ is (5) by its upper bound as given by (6) and require

$$\overline{NCB} + W\sum_i \sum_j \sigma(\widetilde{PFLOAT_{ij}}) + W \sum_i \sum_m \sigma(\widetilde{NFLOAT_{im}}) \geq 0$$

or $$\overline{NCB} \geq |W| \sum_i \sum_j \sigma(\widetilde{PFLOAT_{ij}}) + |W| \sum_i \sum_m \sigma(\widetilde{NFLOAT_{im}}) \quad , \tag{7}$$

it follows that any $\overline{NCB}$ satisfying expression (7) will also satisfy the original probability constraint.

Define

$$PCUSH_{ij} = |W| \left[\frac{\sigma(\widetilde{PFLOAT_{ij}})}{\overline{PFLOAT_{ij}}} \right] \tag{8}$$

$$NCUSH_{im} = |W| \left[\frac{\sigma(\widetilde{NFLOAT_{im}})}{\overline{NFLOAT_{im}}} \right] \tag{9}$$

where the coefficients inside the bracket are simply the coefficients of variation of the float balances. They measure the standard deviation per dollar of expected float balances. These quantities can be measured from an analysis of the variations in item sizes and collection times.

Substituting (8) and (9) into (7) we obtain the desired constraint

$$\widetilde{NCB} \geq \sum_i \sum_j (PCUSH_{ij})(\overline{PFLOAT_{ij}}) + \sum_i \sum_m (NCUSH_{im})(\overline{NFLOAT_{im}}) \; .$$

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